APTITUDE - NUMBER SYSTEM

Numbers

In Decimal number system, there are ten symbols namely 0,1,2,3,4,5,6,7,8 and 9 called digits. A number is denoted by group of these digits called as numerals.

Face Value

Face value of a digit in a numeral is value of the digit itself. For example in 321, face value of 1 is 1, face value of 2 is 2 and face value of 3 is 3.

Place Value

Place value of a digit in a numeral is value of the digit multiplied by 10^n where n starts from 0. For example in 321:

- Place value of 1 = 1 x 10^0 = 1 x 1 = 1
- Place value of 2 = 2 x 10^1 = 2 x 10 = 20
- Place value of 3 = 3 x 10^2 = 3 x 100 = 300

> *0^{th} position digit is called unit digit and is the most commonly used topic in aptitude tests.*

Types of Numbers

1. **Natural Numbers** - n > 0 where n is counting number; [1,2,3...]
2. **Whole Numbers** - n ≥ 0 where n is counting number; [0,1,2,3...].

> *0 is the only whole number which is not a natural number.*
>
> *Every natural number is a whole number.*

3. **Integers** - n ≥ 0 or n ≤ 0 where n is counting number;...,-3,-2,-1,0,1,2,3... are integers.
    - **Positive Integers** - n > 0; [1,2,3...]
    - **Negative Integers** - n < 0; [-1,-2,-3...]
    - **Non-Positive Integers** - n ≤ 0; [0,-1,-2,-3...]
    - **Non-Negative Integers** - n ≥ 0; [0,1,2,3...]

> *0 is neither positive nor negative integer.*

4. **Even Numbers** - n / 2 = 0 where n is counting number; [0,2,4,...]
5. **Odd Numbers** - n / 2 ≠ 0 where n is counting number; [1,3,5,...]

6. **Prime Numbers** - Numbers which is divisible by themselves only apart from 1.

> *1 is not a prime number.*
>
> *To test a number p to be prime, find a whole number k such that k > √p. Get all prime numbers less than or equal to k and divide p with each of these prime numbers. If no number divides p exactly then p is a prime number otherwise it is not a prime number.*

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Example: 191 is prime number or not?
Solution:
Step 1 - 14 > √191
Step 2 - Prime numbers less than 14 are 2,3,5,7,11 and 13.
Step 3 - 191 is not divisible by any above prime number.
Result - 191 is a prime number.

Example: 187 is prime number or not?
Solution:
Step 1 - 14 > √187
Step 2 - Prime numbers less than 14 are 2,3,5,7,11 and 13.
Step 3 - 187 is divisible by 11.
Result - 187 is not a prime number.
```

7. **Composite Numbers** - Non-prime numbers > 1. For example, 4,6,8,9 etc.

> *1 is neither a prime number nor a composite number.*
>
> *2 is the only even prime number.*

8. **Co-Primes Numbers** - Two natural numbers are co-primes if their H.C.F. is 1. For example, (2,3), (4,5) are co-primes.

Divisibility

Following are tips to check divisibility of numbers.

1. **Divisibility by 2** - A number is divisible by 2 if its unit digit is 0,2,4,6 or 8.

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Example: 64578 is divisible by 2 or not?
Solution:
Step 1 - Unit digit is 8.
Result - 64578 is divisible by 2.

Example: 64575 is divisible by 2 or not?
Solution:
Step 1 - Unit digit is 5.
Result - 64575 is not divisible by 2.
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2. **Divisibility by 3** - A number is divisible by 3 if sum of its digits is completely divisible by 3.

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Example: 64578 is divisible by 3 or not?
Solution:
Step 1 - Sum of its digits is 6 + 4 + 5 + 7 + 8 = 30
which is divisible by 3.
Result - 64578 is divisible by 3.

Example: 64576 is divisible by 3 or not?
Solution:
Step 1 - Sum of its digits is 6 + 4 + 5 + 7 + 6 = 28
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which is not divisible by 3.
Result - 64576 is not divisible by 3.
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3. **Divisibility by 4** - A number is divisible by 4 if number formed using its last two digits is completely divisible by 4.

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Example: 64578 is divisible by 4 or not?
Solution:
Step 1 - number formed using its last two digits is 78
which is not divisible by 4.
Result - 64578 is not divisible by 4.

Example: 64580 is divisible by 4 or not?
Solution:
Step 1 - number formed using its last two digits is 80
which is divisible by 4.
Result - 64580 is divisible by 4.
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4. **Divisibility by 5** - A number is divisible by 5 if its unit digit is 0 or 5.

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Example: 64578 is divisible by 5 or not?
Solution:
Step 1 - Unit digit is 8.
Result - 64578 is not divisible by 5.

Example: 64575 is divisible by 5 or not?
Solution:
Step 1 - Unit digit is 5.
Result - 64575 is divisible by 5.
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5. **Divisibility by 6** - A number is divisible by 6 if the number is divisible by both 2 and 3.

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Example: 64578 is divisible by 6 or not?
Solution:
Step 1 - Unit digit is 8. Number is divisible by 2.
Step 2 - Sum of its digits is 6 + 4 + 5 + 7 + 8 = 30
which is divisible by 3.
Result - 64578 is divisible by 6.

Example: 64576 is divisible by 6 or not?
Solution:
Step 1 - Unit digit is 8. Number is divisible by 2.
Step 2 - Sum of its digits is 6 + 4 + 5 + 7 + 6 = 28
which is not divisible by 3.
Result - 64576 is not divisible by 6.
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6. **Divisibility by 8** - A number is divisible by 8 if number formed using its last three digits is completely divisible by 8.

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Example: 64578 is divisible by 8 or not?
Solution:
Step 1 - number formed using its last three digits is 578
which is not divisible by 8.
Result - 64578 is not divisible by 8.

Example: 64576 is divisible by 8 or not?
Solution:
Step 1 - number formed using its last three digits is 576
which is divisible by 8.
Result - 64576 is divisible by 8.
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7. **Divisibility by 9** - A number is divisible by 9 if sum of its digits is completely divisible by 9.

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Example: 64579 is divisible by 9 or not?
Solution:
Step 1 - Sum of its digits is 6 + 4 + 5 + 7 + 9 = 31
which is not divisible by 9.
Result - 64579 is not divisible by 9.

Example: 64575 is divisible by 9 or not?
Solution:
Step 1 - Sum of its digits is 6 + 4 + 5 + 7 + 5 = 27
which is divisible by 9.
Result - 64575 is divisible by 9.
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8. **Divisibility by 10** - A number is divisible by 10 if its unit digit is 0.

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Example: 64575 is divisible by 10 or not?
Solution:
Step 1 - Unit digit is 5.
Result - 64578 is not divisible by 10.

Example: 64570 is divisible by 10 or not?
Solution:
Step 1 - Unit digit is 0.
Result - 64570 is divisible by 10.
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9. **Divisibility by 11** - A number is divisible by 11 if difference between sum of digits at odd places and sum of digits at even places is either 0 or is divisible by 11.

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Example: 64575 is divisible by 11 or not?
Solution:
Step 1 - difference between sum of digits at odd places
and sum of digits at even places = (6+5+5) - (4+7) = 5
which is not divisible by 11.
Result - 64575 is not divisible by 11.

Example: 64075 is divisible by 11 or not?
Solution:
Step 1 - difference between sum of digits at odd places
and sum of digits at even places = (6+0+5) - (4+7) = 0.
Result - 64075 is divisible by 11.
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Tips on Division

1. If a number n is divisible by two co-primes numbers a, b then n is divisible by ab.
2. (a-b) always divides (a^n - b^n) if n is a natural number.
3. (a+b) always divides (a^n - b^n) if n is an even number.
4. (a+b) always divides (a^n + b^n) if n is an odd number.

Division Algorithm

When a number is divided by another number then

Dividend = (Divisor x Quotient) + Reminder

Series

Following are formulaes for basic number series:

1. (1+2+3+...+n) = (1/2)n(n+1)

2. $(1^2+2^2+3^2+...+n^2) = (1/6)n(n+1)(2n+1)$
3. $(1^3+2^3+3^3+...+n^3) = (1/4)n^2(n+1)^2$

Basic Formulaes

These are the basic formulae:

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$(a + b)^2 - (a - b)^2 = 4ab$$

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$(a^2 - b^2) = (a + b)(a - b)$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$$

$$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$$

$$(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$